Elementary particles and typical scales in high energy physics

George Jorjadze

Free University of Tbilisi

Zielona Gora - 23.01.2017

Contents

Review of the main developments in particle physics (PP)

- First unification of interactions
- Special relativity, quantum mechanics
- Four forces of nature, QFT
- The Weinberg-Salam model
- The Standard model
- New unifications

Typical scales in high energy physics (HEP)

- Units in particle physics
- Compton radius
- Bohr radius
- Classical electron radius
- Plank scale

First unification of interactions

- Maxwell's equations

In 1865 Maxwell constructed a consistent set of equations

$$\partial_{[\alpha}F_{\mu\nu]} = 0 \ , \qquad \partial_\mu F^{\mu\nu} = J^\nu \ ,$$

which unify electricity and magnetism.

This elegant unification was not optional:

separate theories of electricity and magnetism would be inconsistent.

About one-hundred years later another fundamental unification between electromagnetic and week interaction occurred.

To understand the significance of this unification it is necessary to review the main developments in physics since 1865.

Special relativity

- Space-time unification

In this theory one finds a conceptual unification of the separate notions of space and time, which is a new recognition of the area where physical phenomena take place.

Newtonian mechanics was replaced by relativistic mechanics

$$L = \frac{m\dot{x}^2}{2} \quad \mapsto \quad L = -mc^2\sqrt{1 - \frac{\dot{x}^2}{c^2}} \ ,$$

where mass and energy are interchangeable

$$E = \frac{mc^2}{\sqrt{1 - \frac{\dot{x}^2}{c^2}}} \ .$$

and one has the dispersion relation

$$E^2 = (mc^2)^2 + (pc)^2 \ .$$

Quantum mechanics

- $\hbar$ and uncertainties

Quantum mechanics is more than a theory. It is a framework for the description of microscopic phenomena.

It gives the rules which must be used to extract physical predictions.

One has quantum observables as operators $\hat{F} = F(\hat{p}, \hat{q})$ constructed from classical ones $F(p, q)$.

In particular,

$$q \mapsto \hat{q} \ , \quad p \mapsto \hat{p} \ , \quad \text{such that} \quad [\hat{q}, \hat{p}] = i\hbar\hat{I} \ .$$

The fundamental constant $\hbar$.

One gets a probabilistic character of observables

$$\bar{F} = \langle\, a|\hat{F}|a\,\rangle \ , \quad \Delta p \cdot \Delta q \geq \frac{\hbar}{2} \ , \quad P_{ab} = |\langle\, a|b\,\rangle|^2 \ .$$

Four fundamental forces

- Interactions through field theory

1. The force of gravity by Einstein's theory of general relativity.
It is a classical theory of the spacetime dynamics

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}\ .$$

Gravitational force arise by the curvature of the dynamical spacetime.

2. The electromagnetic force described by Maxwell's equations.

3. The week force is responsible for nuclear decay.
The recognition of this force came in the middle of twentieth century.

4. The strong, or color, force. This force holds together constituents of proton, neutron, pions, etc. The constituents are called quarks.

One needs Yang-Mills theory for week and strong forces.

The Winberg-Salam model

- Second unification of interactions

It puts together electromagnetism and the week forces and creates electroweek interactions.

This unification was necessary for a consistent theory of week forces.

The theory was initially formulated with four massless particles that carry the forces (gauge bosons).

The spontaneous symmetry breaking with Higgs field gives masses to three of these particles $W_\pm$ and Z_0 by the following type Lagrangian

$$\mathcal{L} = \frac{1}{4} F^2 + |D\phi|^2 - \lambda(|\phi|^2 - a^2)^2 \ ,$$

where F is Yang-Mills field, ϕ is Higgs field and a is a parameter.

Quantum field theory (QFT)

- Particles as quanta of fields

Classical electrodynamics is neither an accurate nor a correct theory for microscopic phenomena.

QFT is a theory of quantized fields.
Quantization turns a classical theory to quantum theory (Dirac).

Examples:
Quantum electrodynamics (QED), where photons appear as the quanta of the electromagnetic field.

The Weingerg-Salam model also became a consistent QFT of electroweek interactions (t'Hooft).

QFT of strong interactions - quantum chromodynamics (QCD).
The carriers of the color force are eight massless particles - (gluons).

The Standard Model (SM)

- 60 'elementary particles'

The electroweek theory together with QCD form SM.

There is some interplay between electroweek sector and the QCD sector because some particles feel both types of forces.
But there is no real unification of week forces and the color forces.

SM summarizes the present knowledge of PP.

12 force carriers: 8 gluons g_a, 3 bosons $W_\pm$, Z_0, photon γ.

12 leptons: (e, ν_e), (μ, ν_μ), (τ, ν_τ)
(together with antiparticles).

36 quarks: (u, d), (c, s), (t, b)
(in 3 colors and together with antiparticles).

Shortcomings of SM

- Gravity?

1. Contains about 20 parameters.

For example the ration of the mass of the muon and the mass of the electron is 207, which has to be taken by hand.

2. It does not include gravity.

The effects of the gravity are quite negligible in PP, however they are crucial in cosmology and in the study of the early universe.

SM is a quantum theory, while general relativity is a classical theory.

There is no consistent quantum theory for general relativity.

Formulating a quantum theory that includes both gravity and other forces is fundamentally necessary.

New unifications

- From particles to strings

Physicists believe that SM is a step towards the complete theory.

Possible directions:

1. Unification of the electroweek and strong forces into a Grand Unified Theory (GUT).

2. Supersymmetry, supergravity.

Supersymmetry relates bosons and fermions.
Matter particles are fermions and carriers of interactions are bosons.
This symmetry unifies matter and forces.
Supergravity is a supersymmetric theory of gravity.

3. Superstring theory.

All particles are combined in one geometrical object - string.
Particles are treated as string excitations.

Units in particle physics

- To use convenient units

$m_e = 9.109 \times 10^{-31}$ kg
$e = -1.602 \times 10^{-19}$ C
$\hbar = 1.055 \times 10^{-34}$ J·s
$c = 299\ 792\ 458$ m/s

In high energy particle physics it is convenient to choose:

$c = 1$ and $\hbar = 1$. They are called natural units.

One gets the dimensions

[velocity]=pure number,
[energy]=[momentum]=[mass],
[length]=[mass]$^{-1}$.

Particle velocity is bounded by $0 \leq v \leq 1$.
Spin becomes integer (for bosons) or half-integer (for fermions).

Mass and length

- Mass $\sim$ length^{-1}

An useful unit of energy is MeV$=10^6$ eV. $m_e \approx 0.5$ MeV$/c^2$.

A typical length scale is fermi:
1 fm$=10^{-13}$ cm, which is the size of a proton.

The dimension of $\hbar c$ in standard units is [energy]$\times$[length]
From the numbers, given above, one gets

$$\hbar c \approx 200 \text{ Mev} \cdot \text{fm}.$$

Therefore, in the natural units

$$1\,\text{fm} \approx 0.5 \times 10^{-2}\,\text{MeV}.$$

Below we describe some dimensional estimates of physical quantities.

Compton radius

- The fine structure constant

$$\alpha = \frac{e^2}{4\pi\hbar c} \approx \frac{1}{137} \ .$$

- Compton wave length

The simplest length-scale associated to a particle of mass m in its rest frame is the Compton radius $r_C = 1/m$.

For the electron

$$r_C = \frac{1}{m_e} \approx \frac{200\text{MeV} \cdot \text{fm}}{0.5\text{Mev}} = 4 \times 10^{-11}\text{cm} \ .$$

The Compton wavelength of a particle is equivalent to the wavelength of a photon whose energy is the same as the mass of the particle

$$r_C = \frac{\hbar}{mc} \ .$$

Bohr radius

- Hydrogen atom

First let us estimate the Bohr radius r_B.

$m_p = 938$ MeV, $m_e = 0.5$ MeV. The reduced mass is

$$\mu = \frac{m_p\, m_e}{m_p + m_e} \approx m_e \ .$$

The radius has to depend on α.
Quantum uncertainty yields $p \approx 1/r$.
The kinetic energy

$$E_k = p^2/2m \approx \frac{1}{2m_e r^2} \ .$$

By virial theorem $E_k = \frac{\alpha}{2r}$, i.e.

$$r_B = \frac{1}{\alpha\, m_e} \approx 0.5 \times 10^{-8}\,\mathrm{cm} \ .$$

More on hydrogen atom

- QED corrections to energy levels

The typical potential energy of the hydrogen atom is then

$$\langle V \rangle \approx V(r_B) = -\frac{\alpha}{r_B} = -m_e \alpha^2 \ .$$

Using again the viral theorem the kinetic energy is

$$E_k = \frac{|V|}{2} \approx \frac{1}{2} m_e \alpha^2 \ .$$

Thus, $v \sim \alpha$ and the approximation of a non-relativistic is consistent. The total energy then is

$$E = -\frac{1}{2} m_e \alpha^2 \approx -13.6 eV \ .$$

QED gives the energy levels

$$E_n = m_e \left[-\frac{\alpha^2}{2n^2} - \frac{\alpha^4}{2n^4} \left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4} \right) + \cdots \right] \ .$$

Classical electron radius

- Electron-photon scattering $e\,\gamma \mapsto e\,\gamma$

The energy of the photon $E = \hbar\omega$ in the natural units is just ω.
Let's consider the case when $\omega \ll m_e$.
In this case we can neglect ω compared to m_e.
We then have one mass-scale m_e.
The cross-section σ has dimensions [length]^2 and $\sigma \sim \alpha^2/m_e^2$.

It is therefore useful to define

$$r_0 = \frac{\alpha}{m_e} \approx 2.8 \times 10^{-13}\text{cm} \; .$$

The exact computation gives the result (Thomson cross-section)

$$\sigma = \frac{8}{3}\,\pi r_0^2 \; .$$

r_0 is called classical electron radius.

The Plank scale

- The Plank mass and the Plank length

From the fundamental constants $\hbar$, c and the Newton constant G_N one gets mass-scale parameter

$$M_{Pl} = \sqrt{\frac{\hbar c}{G_N}} \approx 1.2 \times 10^{19}\,\mathsf{Gev}/c^2 \ .$$

Similarly, the Plank length is defined by

$$L_{Pl} = \sqrt{\frac{\hbar\, G_N}{c^3}} \approx 1.6 \times 10^{-33}\,\mathsf{cm} \ .$$

The Plank units: $M_{Pl} = \hbar = c = 1$.

This units are used in quantum gravity and string theory.